11–4 Irrational Square Roots

Objective: To simplify radicals and to find decimal approximations of irrational square roots.

Vocabulary

Irrational numbers Real numbers that can't be expressed in the form $\frac{a}{b}$, where a and b are integers. Their exact values can't be expressed as either terminating or repeating decimals.

Property

Property of Completeness Every decimal represents a real number, and every real number can be represented by a decimal.

Example 1 Simplify: **a.** $\sqrt{256}$ **b.** $\sqrt{50}$ **c.** $2\sqrt{80}$ **d.** $\sqrt{704}$

Solution

a. $\sqrt{256} = \sqrt{4 \cdot 64}$ Factor within the radical sign.
$= \sqrt{4} \cdot \sqrt{64}$ Use the product property of square roots.
$= 2 \cdot 8$ Simplify.
$= 16$

b. $\sqrt{50} = \sqrt{25 \cdot 2}$
$= \sqrt{25} \cdot \sqrt{2}$
$= 5\sqrt{2}$

c. $2\sqrt{80} = 2\sqrt{16 \cdot 5}$
$= 2 \cdot 4\sqrt{5}$
$= 8\sqrt{5}$

d. $\sqrt{704} = \sqrt{64 \cdot 11}$
$= 8\sqrt{11}$

Simplify.

1. $\sqrt{27}$ **$3\sqrt{3}$**
2. $\sqrt{20}$ **$2\sqrt{5}$**
3. $\sqrt{72}$ **$6\sqrt{2}$**
4. $\sqrt{32}$ **$4\sqrt{2}$**
5. $\sqrt{48}$ **$4\sqrt{3}$**
6. $\sqrt{45}$ **$3\sqrt{5}$**
7. $\sqrt{196}$ **14**
8. $\sqrt{80}$ **$4\sqrt{5}$**
9. $2\sqrt{63}$ **$6\sqrt{7}$**
10. $4\sqrt{98}$ **$28\sqrt{2}$**
11. $7\sqrt{28}$ **$14\sqrt{7}$**
12. $4\sqrt{40}$ **$8\sqrt{10}$**
13. $\sqrt{441}$ **21**
14. $\sqrt{289}$ **17**
15. $3\sqrt{50}$ **$15\sqrt{2}$**
16. $12\sqrt{50}$ **$60\sqrt{2}$**
17. $\sqrt{729}$ **27**
18. $\sqrt{432}$ **$12\sqrt{3}$**
19. $8\sqrt{75}$ **$40\sqrt{3}$**
20. $2\sqrt{90}$ **$6\sqrt{10}$**
21. $\sqrt{147}$ **$7\sqrt{3}$**
22. $\sqrt{288}$ **$12\sqrt{2}$**
23. $\sqrt{4225}$ **65**
24. $5\sqrt{800}$ **$100\sqrt{2}$**
25. $5\sqrt{1025}$ **$25\sqrt{41}$**

Example 2 Approximate $\sqrt{396}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$\sqrt{396} = \sqrt{2^2 \cdot 3^2 \cdot 11}$
$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{11}$
$= 6\sqrt{11}$

From the table: $\sqrt{11} \approx 3.317$
$6\sqrt{11} \approx 6(3.317) \approx 19.902$

Therefore $\sqrt{396} \approx 19.90$.

Example 3 Approximate $\sqrt{0.6}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$\sqrt{0.6} = \frac{\sqrt{60}}{\sqrt{100}} = \frac{\sqrt{60}}{10} \approx \frac{7.746}{10} = 0.7746$

Therefore $\sqrt{0.6} \approx 0.77$.

In Exercises 26–37, use your calculator or the table at the back of the book.

Approximate each square root to the nearest tenth.

26. $\sqrt{600}$ **24.5**
27. $\sqrt{200}$ **14.1**
28. $-\sqrt{800}$ **-28.3**
29. $-\sqrt{500}$ **-22.4**
30. $-\sqrt{2700}$ **-52.0**
31. $-\sqrt{2200}$ **-46.9**
32. $\pm\sqrt{6600}$ **± 81.2**
33. $\pm\sqrt{4800}$ **± 69.3**

Approximate each square root to the nearest hundredth.

34. $\sqrt{56}$ **7.48**
35. $\sqrt{32}$ **5.66**
36. $-\sqrt{0.7}$ **-0.84**
37. $-\sqrt{0.2}$ **-0.45**

Mixed Review Exercises

Find the indicated square roots.

1. $\sqrt{100}$ **10**
2. $-\sqrt{144}$ **-12**
3. $\sqrt{\frac{9}{25}}$ **$\frac{3}{5}$**
4. $-\sqrt{\frac{36}{121}}$ **$-\frac{6}{11}$**
5. $\sqrt{154^2}$ **154**
6. $\sqrt{\left(\frac{2}{5}\right)^2}$ **$\frac{2}{5}$**

Simplify.

7. $(13x)^2$ **$169x^2$**
8. $(2y^3z^6)^2$ **$4y^6z^{12}$**
9. $(x + 2y)^2$ **$x^2 + 4xy + 4y^2$**
10. $[10(a + 1)]^2$ **$100a^2 + 200a + 100$**
11. $(9a^3b^7c)^2$ **$81a^6b^{14}c^2$**
12. $(4z^2 + 3y^3)(4z^2 - 3y^3)$ **$16z^4 - 9y^6$**